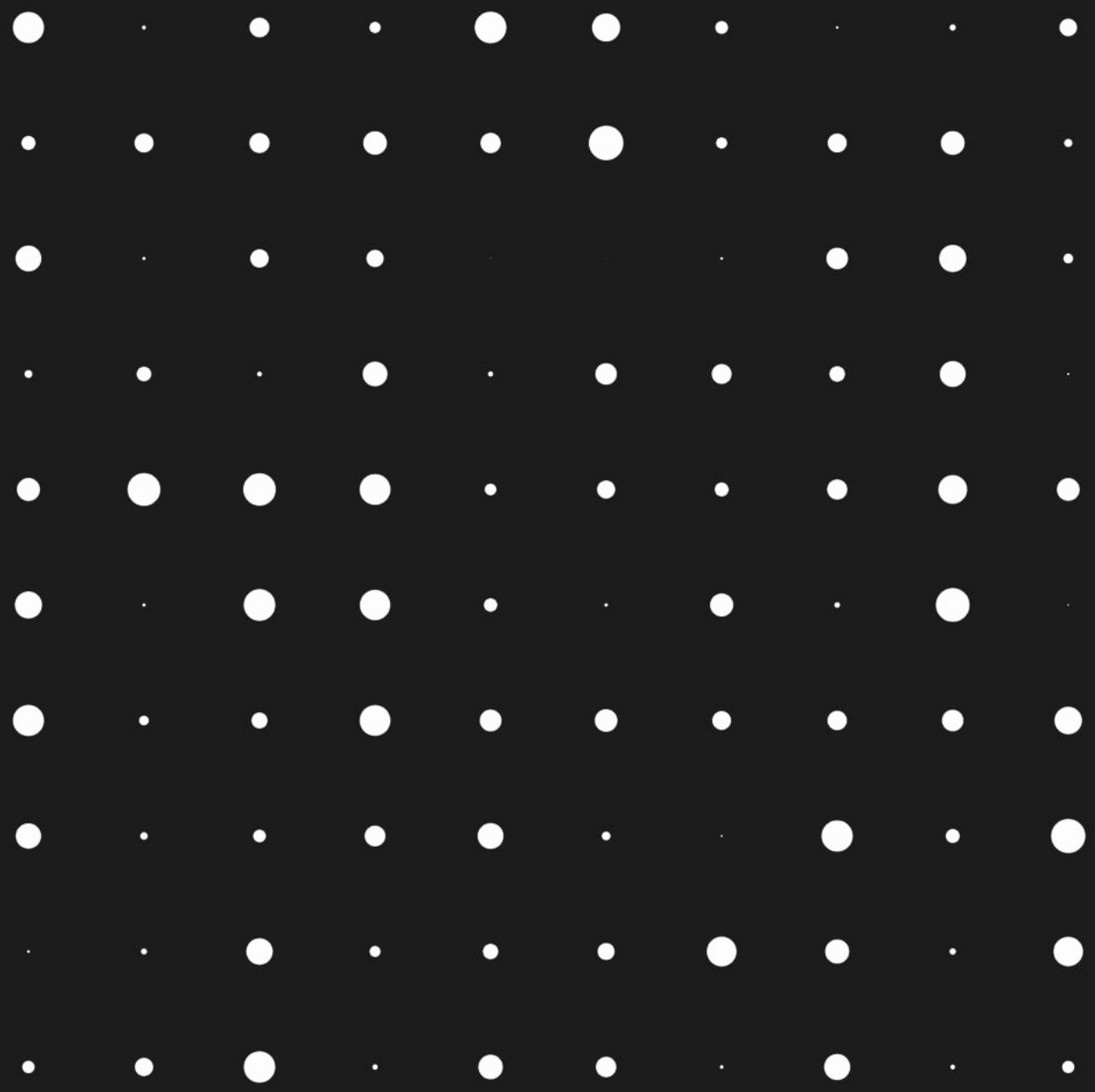


FC 1110 - Quantitative Reasoning & Mathematical Thinking

Monsoon 2024 • Question Bank

Department of Computer Science
Ashoka University

Instructors : **Prof. Bhargab Bikram Bhattacharya**
Teaching Fellow : **Shubhajit Dey**

Department of Computer Science
Ashoka University

FC-1110 QRMT - Monsoon 2024
QUESTION BANK

Instructors : Prof. Bhargab Bikram Bhattacharya
Teaching Fellow : Shubhajit Dey

DISCLAIMER

This document serves as a combined version of all the formal assessments (quizzes + mid term + end term), during the span of the **FC1110 - Quantitative Reasoning & Mathematical Thinking** course in the Monsoon 2024 semester at Ashoka University.

The course was designed as an open-ended foundational course to give undergrad freshmen as taste of the basics of computer science and algorithmic thinking. The goal was to present clarity, and help students to take an informed decision about opting for a major/minor/-concentration in CS.

Furthermore, note that this document is just for academic assistance to whomever needs it and any instance of typos and conflicting content is requested to be reported over an email to shubhajit.acad[@]icloud[dot]com.

[**Declaration**] The cover design is due to Processing Foundation (2019 project), link.

YOU NEED TO COMBINE YOUR DREAM AS A KID, YOUR OBSESSION AS AN ADULT, WITH SOMETHING THE WORLD DESPERATELY NEEDS.

EVERYTHING ELSE IS A DISTRACTION.

FIGURING THIS OUT IS THE HARD PART, AND THE FUN PART.

Sample Solution

Ashoka University

Monsoon 2024

FC-0306-4

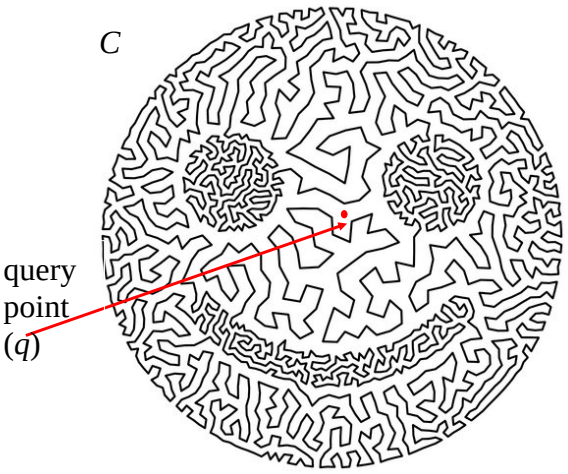
Quantitative Reasoning and Mathematical Thinking

Date: 04 September 2024 **Total points: 50 (**= 40 + 10 (Bonus)); Credit: 5%
Quiz # 1 **Time:** 5:15 PM - 6:00 PM

Name: …… **Ashoka ID #** ……

Instructions (*Read carefully*)**:** This is an MCQ or short-answer type, OPEN-BOOK, OPEN-NOTES, quiz. For each **MCQ**, please choose one answer from the given choices. Each *correct answer* to an MCQ will fetch 4.0 points, *incorrect answer* will contribute 0 point, and *no answer* (omission) leads to 1.0 point. Thus, if you have doubt, it may be beneficial to *skip* an MCQ. For a short-answer type question, its components, if any, will be graded proportionately. All questions carry **4 points**, each. This quiz has **ten** questions and **two** pages.

Submission of answers: Please write **your name, ID**, and your **answers** on this Test Paper itself.



1. (a) Consider the curve *C* shown on the left. Name three properties that are satisfied by *C*.

(i) closed and finite;

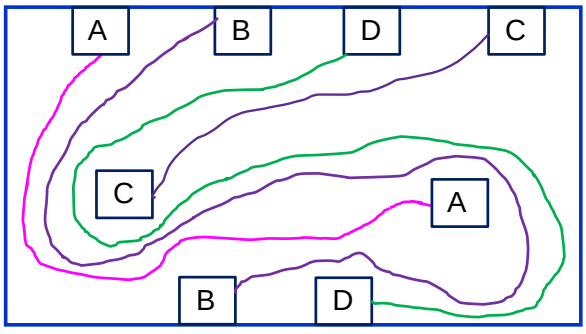
(ii) continuous;

(iii) non-self-intersecting

(b) Consider the query point *q* as shown. Describe a brief procedure to ascertain whether or not *q* lies in the interior of C.
Procedure: Shoot an infinite ray from the query point *q* to any direction; count the number (*N*) of intersections with *C*. If *N* is even, *q* is in the exterior of *C*, else it is in the interior. In this example, it is in the *exterior*.

2. In the figure below, a rectangular bounding box with terminals A, B, C, D is shown. The objective is to draw lines/curves that connect (A to A); (B to B); (C to C); (D to D) such that no connecting lines/curves intersect, and none of them should touch or cross the boundary of the bounding box. Show a solution, or prove that it is impossible to solve this configuration.

A solution is shown here.



3. *Pallavi* was born in the year 2002 and now she is doing Major in CS. In college, friends call her *Pali*. What is her date of birth? Gear up your imagination and justify your answer.
Since she is in CS and friends call her *Pali*, it is indicative of some pattern, which may be a palindrome. Indeed, there is one: her D-o-B (*ddmmyyyy*) is 20022002, i.e., 20 Feb 2002.

4. Eleven integers have been chosen randomly from 1, 2, 3, …., 20; show by applying pigeon-hole principle, that two of them must be consecutive.
Proof: If two integers are consecutive, they must be either (even, odd) or (odd, even). There are ten even integers and ten odd integers among 1, 2, 3, …., 20. Since we have to choose eleven integers (pigeons) from ten even/odd integers (holes), by PHP, there must be at least one even-odd or odd-even consecutive pair.

5. The total number of ways you can rearrange the letters in "ARRANGE" so that all the vowels always appear together, is (*choose one*):

A. 5040; B. 1260; C. 180; D. 60; E. None of these.

6. The houses in a locality are numbered with two different letters (out of A, B, C, …, Z) followed by two different digits (out of 0, 1, 2, …, 9). The number of possible house number-plates, is (*choose one*):

A. 67600; B. 58500; C. 7760; D. 7400; E. None of these.

7. Which statement among the following, is TRUE (*choose one*)? $C(n, r)$ denotes the number of r-combinations out of n objects.

(i) $C(15, 5) = 2 \times C(15, 10)$; (ii) $C(15, 5) = (1/3) \times C(15, 10)$; (iii) $C(15, 5) = C(15, 10)$;

(iv) $C(15, 5) = (½) \times C(15, 10)$; (v) None of these.

8. In a test, Rimi has to answer exactly 10 questions out of 15 questions with the constraint that she has to answer 2 questions from the first 5 questions, and 8 from the remaining 10 questions. The number of choices Rimi can have, is (*choose one*):

A. 3003; B. 450; C. 55; D. 1287; E. None of these.

9. We have three dice colored red, blue, and green, which are being simultaneously rolled. What is the number of outcomes such that all values are distinct?

A. 216; B. 210; C. 180; D. 120; E. none of these

10. A binary string comprises 0's and 1's only. We consider all 8-bit binary strings, each of which contains exactly five 0's and three 1's. For example, 11100000 is a valid string whereas 10101110 is not. The number of such strings is (choose one):

A. 6; B. 32; C. 128; D. 256; E. None of these

End of Quiz. Good Luck!

Model Solutions

Ashoka University

Monsoon 2024

FC-0306-4

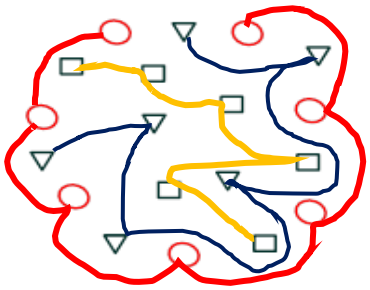
Quantitative Reasoning and Mathematical Thinking

Date: 18 September 2024 **Total points: 50 (**= 48 + 2 (Bonus)); Credit: 5%
Quiz # 2 **Time:** 5:00 PM - 6:00 PM

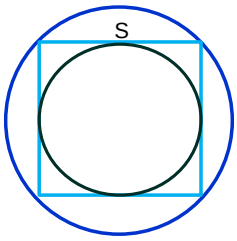
Name: …… **Ashoka ID #** ……

Instructions (*Read carefully*)**:** This is an MCQ or short-answer type, OPEN-BOOK, OPEN-NOTES, quiz. For each **MCQ**, please choose one answer from the given choices. Each *correct answer* to an MCQ will fetch 4.0 points, *incorrect answer* will contribute 0 point, and *no answer* (omission) leads to 1.0 point. For a short-answer type question, its components, if any, will be graded proportionately. All questions carry **4 points**, each. This quiz has **twelve** questions and **two** pages.

Submission of answers: Please write **your name, ID**, and your **answers** on this Test Paper itself.



1. Consider three classes of objects: disks, rectangles, and triangles, as shown in the figure left. All objects belonging to the same class are to be interconnected together using straight or curvy lines. The goal is to interconnect all objects in each class, without creating any crossover of curves. Show a solution or argue that it is impossible.
One solution is shown here.



2. In the figure on the left, S denotes a square. S is inscribed by the inner circle and circumscribed by the outer circle as shown. The ratio of the circumference of the outer circle to that of the inner circle is (*fill-up-the gap*): (√2:1) .

3. There are three Presidential candidates *X*, *Y*, *Z*, and 100 voters to cast votes. Every voter will cast *exactly one vote* either to *X* or *Y*, or to *Z*. For example, {*X* = 50, *Y* = 48, *Z* = 2} is a valid vote distribution. The total number of election outcomes in terms of distribution of votes, is (*choose one*):

A. 3^{100} ; B. 100^3 ; C. 5151; D. 161700; E. None of these.

4. The area of circular-coin *A* is four times that of circular-coin *B*. Both *A* and *B* lie on the plane in non-overlapping fashion and touch each other on their boundaries. Starting from an initial position where *B* touches *A*, coin *B rolls around* coin *A* and returns to its starting point without slipping. The number of rotations completed by coin *B* is (*choose one*):

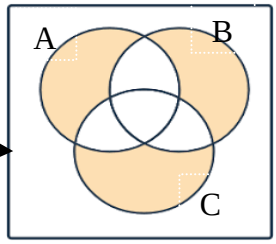
A. 1; B. 3 ; C. 4; D. 5; E. None of these.

5. *A* is travelling from Delhi to Dehradun by car with the average speed of 80 *km/hr*, and *B* is returning from Dehradun to Delhi with the average speed of 60 *km/hr*, along the same road. The distance from Delhi to Dehradun is 280 *km*, and both *A* and *B* started their journey simultaneously at 8 AM, and they are travelling non-stop. The time when *A* and *B* would meet is (*fill-up the gap*):10 AM.

6. Let X be a set $\{\varphi, \{\varphi\}, \{\varphi, \{\varphi\}\}\}$. Write down the power-set $P(X)$ of X:

$P(X) = \{\varphi, \{\varphi\}, \{\{\varphi\}\}, \{\{\varphi, \{\varphi\}\}\}, \{\varphi, \{\varphi\}\}, \{\varphi, \{\varphi, \{\varphi\}\}\}, \{\{\varphi\}, \{\varphi, \{\varphi\}\}\}, \{\varphi, \{\varphi\}, \{\varphi, \{\varphi\}\}\}\}$

7. Write down the set expression for the *shaded area* in terms of A, B, C:



$(A \cap (B \cup C)^C) \cup (B \cap (A \cup C)^C) \cup (C \cap (A \cup B)^C)$, or
$(A \setminus (B \cup C)) \cup (B \setminus (A \cup C)) \cup (C \setminus (A \cup B))$,

There are other valid solutions. Kindly check while grading.

8. In DS of QRMT, there are 10 students and the teacher wants to make one team that comprises at least two students. The number of the ways the team can be formed (choose one):

A. 45; B. 100 ; C. 1013; D. 1024; E. None of these.

9. Assume A and B are two sets such that no one is a subset of the other, and $A \cap B \neq \varphi$. Simplify $X = (B^C \cup A) \cap (A \setminus B)$

$= A \setminus B$ or
$= A \cap B^C$

10. Encircle the numbers below which are rational:

A. $(\sqrt{5})^{\sqrt{2}}$; B. 10.666….∞; C. $\sqrt{(2.56)}$; D. $\sqrt{(49.16)}$; E. π

11. Suppose $\mathbf{N} = \{1, 2, 3, \ldots\}$ is the universal set, and $A = \{n \mid n \leq 6\}$, $B = \{n \mid 4 \leq n \leq 9\}$, and $C = \{1, 3, 5, 7, 9\}$. What is $A \cap (B \oplus C)$?

$A \cap (B \oplus C) = \{1, 3, 4, 6\}$

12. In a batch of 200 students, 100 students took the course on Modern History, 90 students took the course on Indian Philosophy, and 20 students took both the courses. The number of students who neither took Modern History nor Indian Philosophy is (*choose one*):

A. 10; B. 30 ; C. 70; D. 80; E. None of these.

End of Quiz. Good Luck!

Ashoka University

Monsoon 2024

FC-0306-4

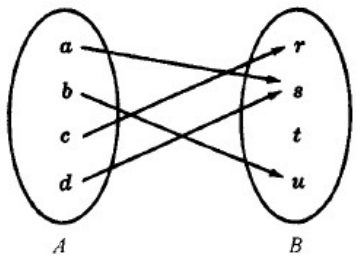
Quantitative Reasoning and Mathematical Thinking

Date: 23 October 2024 **Total points: 50** (= 48 + 2 (Bonus)); Credit: 5%
Quiz # 3 **Time:** 5:00 PM - 6:00 PM

Name: ..... **Ashoka ID #** .....

Instructions (*Read carefully*)**:** This is an MCQ or short-answer type, OPEN-BOOK, OPEN-NOTES, quiz. For each **MCQ**, please choose one answer from the given choices. Each *correct answer* to an MCQ will fetch 4.0 points, *incorrect answer* will contribute 0 point, and *no answer* (omission) leads to 1.0 point. For a short-answer type question, its components, if any, will be graded proportionately. All questions carry **4 points**, each. This quiz has **twelve** questions and **two** pages.

Submission of answers: Please write **your name, ID**, and your **answers** on this Test Paper itself.



1. Let f denote a function from set A to set B, i.e., $f: A \to B$, as shown in the figure on the left. Choose the correct option:

A. f is not a function; B. f is a one-to-one function;
C. f is an onto function; D. f is a bijective function;
E. None of these.

2. Among the following statements, list the correct ones:

A. The set of all prime numbers is uncountably infinite;
B. The set of all integers, including positives, negatives, and zero, is countably infinite;
C. The set of all real numbers is countably infinite;
D. The set of all rational numbers is countably infinite;
E. There is a bijection between the set of all positive even integers, and all positive odd integers.

The correct statements are (fill-up the gaps): B, D, E

3. Let $Z = (-\infty, \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots, +\infty)$, and let $f: Z \to Z$ defined by $f(x) = 2x - 3$.
Is f bijective? If so, prove it; otherwise give a counter-example.
Solution: We show that f is not onto. To prove it, let $y = f(x) = 2x - 3$. Since $y \in Z$, let us assume y be equal to any even integer, say 2. Then $x = 5/2 = 2.5 \notin Z$. So, the pre-image of $f(x) = 2$, does not exist. Hence, f is not onto, and hence, it is not bijective.

4. Let $f: R \to R$, $f(x) = x^2$, where R is the set of all real numbers. Comment on whether f is one-to-one or onto.
Solution: $f(x)$ is NOT one-to-one, because both $x = 2$, $x = -2$, belong to R, and they are mapped to the same value (4) in R. Also, it is NOT onto; to prove this, assume $y = f(x) = x^2 = -2$; Note that -2 belongs to R, but its corresponding $x = \sqrt{(-2)}$ is an imaginary number, and hence does not belong to R.

5. Express 1980 as a product of prime-powers, as stated in the Fundamental Theorem of Arithmetic:
$1980 = 2^2 \times 3^2 \times 5 \times 11$ (Answer).

6. Rimi has a stock on Rs. 2/- notes and Rs. 5/- notes, only. Show, using the principle of mathematical induction, that Rimi can dispense any amount of money, say Rs. x, where x, is a positive integer ≥ 4, using these denominations.

Solution: *Basis:* For Rs. $x = 4$, it is true because $x = 2 + 2$;
Induction Hypothesis: Let us assume that the proposition is true for Rs. $k > 4$. Hence, k can be dispensed with only Rs. 2/- and Rs. 5/- notes.
Case 1: If k comprises at least one 5-rupee note, replace it with three 2-rupee notes. Hence, it is true for Rs. $k + 1$; *Case* 2: k comprises only 2-rupee notes. Since, $k > 4$, it must have at least two 2-rupee notes. Replace two 2-rupee notes by one 5-rupee note. Thus, it is true for Rs. k +1 as well. QED

7. Prove, using the principle of mathematical induction: $n^3 + 2n$ is divisible by 3 for all $n \in N$, $n \geq 1$.
Solution: *Basis:* For $n = 1$, $n^3 + 2n = 3$, which is divisible by 3. Hence, the proposition is true.
Induction hypothesis: Assume it is true for $n = k$ for some $k \in N$, $k > 1$. Hence, $k^3 + 2k$ is divisible by 3.
We will now show that the proposition is also true for $n = k + 1$.
Now, $(k + 1)^3 + 2(k + 1) = (k^3 + 3k^2 + 3k + 1) + 2k + 2 = (k^3 + 2k) + 3k^2 + 3k + 3 =$
$(k^3 + 2k) + 3(k^2 + k + 1)$; the first term is divisible by 3 because of induction hypothesis, and the second term, being a multiple of 3, is divisible by 3. Hence, the proposition is true for $n = k + 1$. QED

8. Given a 5 litre can (A), a 7 litre can (B), a large tank of water, and an empty bucket, how can we pour exactly 1 litre of water into the bucket? Note that there is no volume-markers on the cans.
Solution: Note that gcd(5, 7) = 1 = (3 × 5) – (2 × 7); so, the following procedure will work:

1. Fill up Can A and pour into Can B; 2. Fill up Can A and pour into Can B until it is full (Can A now holds 3 litre of water; 3. Drain Can B fully; 4. Pour the remaining amount of water in Can A to Can B (Can A is now empty, and Can B has 3 litre of water); 5. Fill up Can A and pour into Can B until it is full; 6. Can A now contains only one litre of water, which is poured into the bucket.

9. Compute $gcd(256, 48)$ using the binary method.
Solution: $gcd(256, 48) = 2 \times gcd(128, 24) = 2 \times 2 \times gcd(64, 12) = 2 \times 2 \times 2 \times gcd(32, 6)$
$= 2 \times 2 \times 2 \times 2 \times gcd(16, 3) = 16 \times gcd(8, 3) = 16 \times gcd(4, 3) = 16 \times gcd(2, 3) = 16 \times gcd(1, 2)$
$= 16 \times 1 =$ 16 (Answer).

10. Prove, using set theory, that: 2 + 3 = 5.
Solution: We have the following axioms: 1. a + 0 = a; 2. a + S(b) = S(a +b),
where a, b ∈ W = {0, 1, 2, 3, 4, ……. ∞ }, and where S denotes "successor".
Now, 2 + 3 = 2 + S(2) = S(2 + 2) = S(2 + S(1)) = S(S(2 + 1)) = S(S(2 + S(0))) = S(S(S(2 + 0))) =
= S(S(S(2))) = S(S(3)) = S(4) = 5. QED

11. We have a land of size 2100 meter ×1300 meter, which will be fully sold as a number of square plots of different sizes, but the seller wants to minimize the number of such plots. The orientation of each of the square plots is parallel to the rectangular boundary. The number of such plots will be (choose one):

A. 273; B. 21; C. 13; D. 7; E. None of these

12. We have an array of 100 integers, which are being sorted in non-descending order using the bubble-sort algorithm. The number of comparisons needed to complete the task is (*choose one*):

A. 4950; B. 4900 ; C. 5050; D. 200; E. None of these.

End of Quiz. Good Luck!

Ashoka University

Monsoon 2024

FC-0306-4

Quantitative Reasoning and Mathematical Thinking

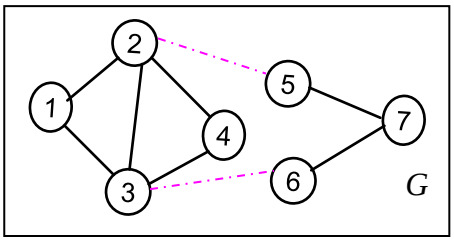
Date: 13 November 2024 **Total points: 50 (**= 48 + 2 (Bonus)); Credit: 5%

Quiz # 4 Model Solution **Time:** 4:50 PM - 6:10 PM

Name: …… **Ashoka ID #** ……

Instructions (*Read carefully*)**:** This is a subjective type, OPEN-BOOK, OPEN-NOTES, quiz. All questions carry **12 points**, each. This quiz has **four** questions and **two** pages.

Submission of answers: Please write **your name** and **ID**, and your answers on this test itself.



1. Consider the graph G comprising seven vertices as shown on the left. Note that G is disconnected because one cannot go from vertex 4 to vertex 5 following a sequence of edges.

(i) Is G a simple graph? If so, why?

(ii) Write down the adjacency matrix $M(G)$ of G;

(iii) Can you identify any pattern in $M(G)$ that tells you that G is disconnected? Justify your answer.

(iv) Some graphs can be drawn without lifting pencil so that each edge is traced exactly once, and we return to the vertex from where we started. Is G traceable in that fashion? If not, add *minimum* number of edges in G between some pairs of vertices so that the modified graph becomes a simple graph, connected, and supports the above-mentioned pencil drawing. Show the modified graph. (1 + 3 + 3 + (1+ 4))

Answer: (i) Note that G does not have any self-loop or parallel edges. Thus, G is simple.

(ii) The adjacency matrix $M(G)$ is given below:

	1	2	3	4	5	6	7
1	0	1	1	0	0	0	0
2	1	0	1	1	0	0	0
3	1	1	0	1	0	0	0
4	0	1	1	0	0	0	0
5	0	0	0	0	0	0	1
6	0	0	0	0	0	0	1
7	0	0	0	0	1	1	0

$M(G)$

(iii) In $M(G)$, the sub-matrices shown in red-fonts comprise all 0's, and they indicate that there is no edge between the subgraph on vertices (1, 2, 3, 4) and the subgraph on vertices (5, 6, 7). Thus, G is disconnected.

(iv) Note that G is not an even graph because vertices 2, 3, 5, 6 have odd degree. Hence, G is not Eulerian, i.e., it is not traceable without lifting pencil and returning to the starting vertex. To make G Eulerian, we need to make all degrees even. Thus, we put an edge between (2, 5) and (3, 6) to make G even (shown as dotted red edges). The resulting graph is simple, connected, and admits a Eulerian cycle, as it is an even graph. Note that putting an edge between (2, 3) and (5, 6) will make G even but the resulting graph would become non-simple and disconnected, and thus it will not work.

2. (a) The following array of numbers is being sorted in increasing order using the insertion-sort algorithm, and the *current status* is given below. By an "iteration", we mean the work necessary to place a particular number to its designated place in the final sorted sequence.

left | *right*

2, 4, 25, 32, 46, 51 | 26, 35, 47, 3, 5, 10, 52, 1, 92, 33

(i) How many additional iterations will be needed to complete sorting? Just write the number.

(ii) What will be status of the array in the next iteration?

Answer: (i) On the right side, we have ten numbers; hence, 10 more iterations are needed.

(ii) The array status in the next iteration: 2, 4, 25, 26, 32, 46, 51 | 35, 47, 3, 5, 10, 52, 1, 92, 33

(b) Four frogs labelled 1, 2, 3, 4 are sitting in cells in a row, the initial configuration of which is shown below. The slot marked as "*" indicates a vacant cell. A frog can only jump forward or backward over exactly one neighboring frog to reach the next cell, provided it is vacant; a frog also can slide into a neighboring cell, left or right, if it is vacant. Only one frog can move at a time. Show the sequence of moves by the frogs so that the final configuration is reached. ((2 + 3) + 7))

Initial configuration: 2 3 4 * 1
Final configuration: * 1 2 3 4

Answer:

2	3	4	*	1	(Initial)
2	3	4	1	*	
2	3	*	1	4	
2	3	1	*	4	
2	*	1	3	4	
2	1	*	3	4	
*	1	2	3	4	(Final)

3. A computing device can multiply only two numbers at a time. Keeping this fact in mind, we want to compute z^{73}, for some $z \in R$, using the binary-based method.
(i) Show the sequence of multiplication steps.
(ii) How may multiplication operations will be needed? (10 +2)

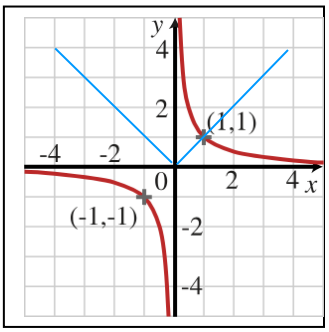
Answer: (i) The binary representation of 73 is 1 0 0 1 0 0 1, i.e., 73 = 64 + 8 + 1. The sequence of multiplications will be:

$$z \to z^2 \to z^4 \to z^8 \to z^{16} \to z^{32} \to z^{64} \to z^{72} \to z^{73}$$
$$z \quad z^2 \quad z^4 \quad z^8 \quad z^{16} \quad z^{32} \quad z^8 \quad z$$

(ii) The required number of multiplications is eight (8).

4. (a) Shreya has put Rs. 10 lakh in bank fixed deposit with 12% interest per annum (nominal rate), compounded quarterly. What effective rate of interest will she be earning?
Answer:
(a) The effective rate of interest = $(1 + 0.12/4)^4 - 1 = (1 + 0.03)^4 - 1 \approx 1.1255 - 1 = 0.1255 = 12.55\%$

(b) Given two equations: $y = 1/x$, and $y = |x|$, solve for x and y by drawing graphical plots of them. Justify your answer. (6 + 6)



Answer: The plots of $y = 1/x$, and $y = |x|$ are shown on the left. Both functions pass through the point (1, 1). Thus, these two functions intersect at (1, 1), and hence, the solutions are: $x = 1$, $y = 1$. From the nature of these two plots, it is clear that this is the only solution.

End of Quz. Good Luck!

Ashoka University

Monsoon 2024

FC-0306-4

Quantitative Reasoning and Mathematical Thinking

Date: 20 November 2024 **Total points: 50 (**= 48 + 2 (Bonus)); Credit: 5%

Quiz # 5 Sample Solution **Time:** 4:50 PM - 6:10 PM

Name: ..... **Ashoka ID #** .....

Instructions (*Read carefully*)**:** This is a subjective type, OPEN-BOOK, OPEN-NOTES, quiz. All questions carry **12 points**, each. This quiz has **four** questions and **two** pages.

Submission of answers: Please write **your name** and **ID**, and your answers on this test itself.

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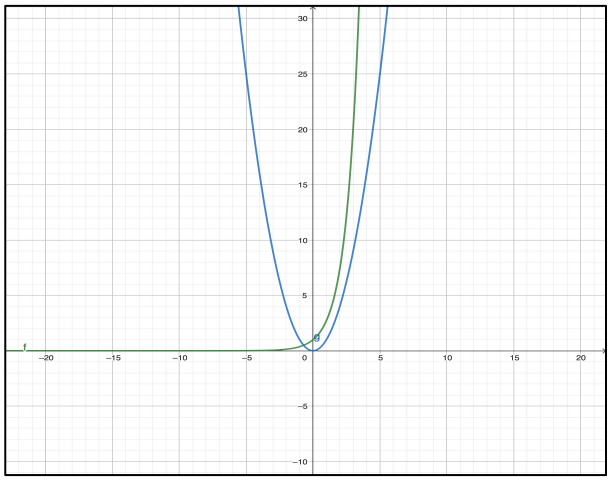
1. Consider the equation : $e^x - x^2 = 0$

(a) Draw the graphical plots of the exponential function and the parabola involved in this equation, and use the graphical plots to tell the number of real solutions of this given equation. Justify your answer properly.



ANSWER : **one.**

In this figure, the green plot is for the exponential and the blue plot is for the parabola. See that these two plots only intersect once, somewhere in the region of negative X-axis. In the region of positive X-axis, the plots slowly diverge and they never meet!

Since the given equation is equivalent to $e^x = x^2$, we can infer from this that the required number of real solution is **one**.

Note for Math, CS and Physics enthusiasts, without the usage of an online Graphical Calculator the answer to this question is very counter intuitive, hence making the question a bit above the intermediate level. **Fact**, this question is from the 2019 ISI B.math entrance examination!

(b) We know that $y = e^x$ is an exponential and $y = x^2$ is a polynomial. But what about their difference, is their difference going to be a polynomial or an exponential? Give one-worded answer.

(10 + 2)

ANSWER : **exponential**.

Continuing from the discussion above, the fact that the plots diverge from each other in the region of positive X-axis as x value increases, implies that the exponential is dominant over the quadratic here. **Fact**, there always exists a point after which exponentials are dominant over all polynomials in general!

2. Construct the Truth Table for the following compound statements, where p and q are individual statements :

(a) $(p \vee \neg q) \implies (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \implies (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

(b) $(p \implies q) \wedge (q \implies p)$ A quick trivia, this is also known as the *Biconditional Implication* or the *if and only iff* statement, denoted as *iff* or with the symbol $\iff$.

(6 + 6)

p	q	$p \implies q$	$q \implies p$	$(p \implies q) \wedge (q \implies p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

3. **Fuzzy logic** is used in artificial intelligence. In fuzzy logic, a statement has a truth value that is a number between 0 and 1, inclusive. A statement with a truth value of 0 is false and one with a truth value of 1 is true. Truth values that are between 0 and 1 indicate varying degrees of truth. For instance, the truth value 0.8 can be assigned to the statement "Fred is happy," because Fred is happy most of the time, and the truth value 0.4 can be assigned to the statement "John is happy," because John is happy slightly less than half the time.

The truth value of the negation of a statement in fuzzy logic is 1 minus the truth value of the statement. The truth value of the AND operator $\wedge$ between two statement in fuzzy logic is the minimum of the truth values of the two statements. What are the truth values of the following statements :

(a) "Fred and John are happy" (b) "Neither Fred Nor John is happy"

(6 + 6)

SOLUTION : Rename the statements as **p** : **'**Fred is happy' and **q** : 'John is happy'. Set any real number between 0 and 1 as the truth values for **p** and **q**. Say truth value for **p** is 0.8 and the truth value for **q** is 0.4 .

(a) This represents **p AND q**, and using the above given rules of
(b) AND operator in fuzzy logic we get the truth value to be **0.4** (the minimum of the two values).

(c) This represents **-p AND -q**, and the truth values of **-p** and **-q** here are 0.2 and 0.6 respectively. Once again using the rules for AND operator we get the required truth value to be **0.2** .

Note for Applied Math and Theoretical CS enthusiasts, Fuzzy Logic forms a major part of a well celebrated research field called Control Theory.

4. A bank sanctioned a loan of INR **P** each to both Sita and Gita, for two years at an interest rate of 5% per annum to each of them. The only catch was that, loan for Sita was at a simple interest scheme whereas for Gita it was at a compounded scheme (compounded annually). After the tenure time, the difference of the total amount repaid by them to bank was INR 25. What was the principal amount **P**, loaned by the bank?

(12)

ANSWER : **INR 10,000**.
Since under same setup compound interest is dominant over simple interest, we solve for the following equation, where r = 5 and t = 2.

$$P\left[(1+\frac{r}{100})^t - (1+\frac{rt}{100})\right] = 25$$

Note for MBA enthusiasts, for t = 2, this setup of 'Interest Problem' has a standard and simpler form, given as follows, which often comes handy for examinations like CAT.

$$\frac{Pr^2}{100^2} = \text{amount difference.}$$

End of Quiz. Good Luck!

Ashoka University

Monsoon 2024

FC-0306-4

Quantitative Reasoning and Mathematical Thinking

Date: 29 November 2024 **Total points: 50 (**= 48 + 2 (Bonus)); Credit: 5%

Quiz # 6 Model Solution **Time:** 4:50 PM - 6:10 PM

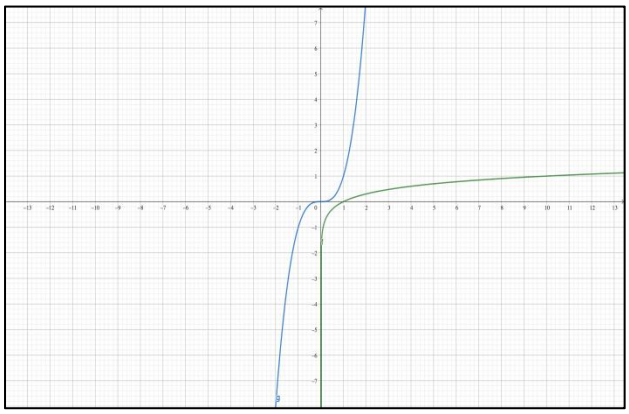
Name: …… **Ashoka ID #** ……

Instructions (*Read carefully*)**:** This is a subjective type, OPEN-BOOK, OPEN-NOTES, quiz. All questions carry **12 points**, each. This quiz has **four** questions and **one** page.

Submission of answers: Please write **your name** and **ID**, and your answers neatly on A4 sized sheets. Scan the sheets into a single **PDF** file and send it over via email before the end time. Please respect the timings, **no late submissions will be graded**.

1. Consider the equation : $\log x - x^3 = 0$

(a) Draw the graphical plots of the logarithmic function and the polynomial involved in this equation, and use the graphical plots to tell the number of real solutions of this given equation. Justify your answer properly.



ANSWER : no real solutions.

In this figure, the green plot is for the logarithm and the blue plot is for the cubic polynomial. See that these two plots do not intersect, not even once. Growth of the cubic polynomial is too much as compared with the logarithm, which guarantees that they never intersect each other in the positive X-axis. Further, in the negative X-axis the the logarithm never exists so no chance of intersection there. This guarantees no intersection.

As usual, the given equation is equivalent to $\log x = x^3$, hence we can infer from this that the required number of real solution is **zero**.

(b) We know that $y = \log x$ is an logarithmic function and $y = x^3$ is a polynomial. But what about their difference, which one of these two is their difference going to behave like in the first quadrant? Give one-worded answer. (10 + 2)

ANSWER : the cubic polynomial.
Hint, look at the graph of $y = x^3 - \log x$.

2. **Fuzzy logic** is used in artificial intelligence. In fuzzy logic, a statement has a truth value that is a number between 0 and 1, inclusive. A statement with a truth value of 0 is false and one with a truth value of 1 is true. Truth values that are between 0 and 1 indicate varying degrees of truth. For instance, the truth value 0.9 can be assigned to the statement "Fred is happy," because Fred is happy most of the time, and the truth value 0.2 can be assigned to the statement "John is happy," because John is happy slightly less than half the time.

The truth value of the negation of a statement in fuzzy logic is 1 minus the truth value of the statement. The truth value of the OR operator $\vee$ between two statement in fuzzy logic is the maximum of the truth values of the two statements. What are the truth values of the following statements.

(a) "Fred or John is happy" (b) "Neither Fred Nor John is happy"

(6 + 6)

SOLUTION : Rename the statements as **p** : **'**Fred is happy' and **q** : 'John is happy'. Set any real number between 0 and 1 as the truth values for **p** and **q**. Say truth value for **p** is 0.9 and the truth value for **q** is 0.2 .

(a) This represents **p OR q**, and using the above given rules of OR operator in fuzzy logic we get the truth value to be **0.9** (the maximum of the two values).

(b) This represents **-p AND -q**, equivalent to **-(p OR q)** due to the De Morgan's Law. Continuing from part (a) the truth value of -(**p OR q**) here is 1 - 0.9 = **0.1**. The required truth value to be **0.1** .

3. Suppose you are given with a collection of sets $A_1, A_2, \ldots, A_5$. The intersection graph of this collection of sets is the graph that has a vertex for each of this sets and has an edge connecting the vertices representing two distinct sets if these two sets have an non-empty intersection.

$A_1 = \{0, 2, 4, 6, 8\}$, $A_2 = \{0, 1, 2, 3, 4\}$,
$A_3 = \{1, 3, 5, 7, 9\}$, $A_4 = \{5, 6, 7, 8, 9\}$,
$A_5 = \{0, 1, 8, 9\}$

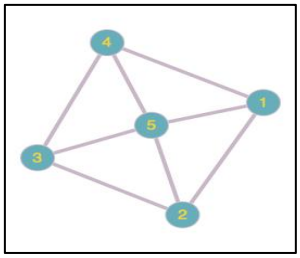
(a) For this given five sets, firstly draw the intersection graph providing necessary calculations supporting your drawing.
(b) Is this a complete graph?
(c) Write the adjacency matrix for this graph.

(5 + 1 + 6)

SOLUTION

(a) Check all possible pairwise intersections to get the required number of edges. Here, adjacency relationships are as follows:

- A_1 is connected to A_2, A_4, and A_5.
- A_2 is connected to A_1, A_3, and A_5.
- A_3 is connected to A_2, A_4, and A_5.
- A_4 is connected to A_1, A_3, and A_5.
- A_5 is connected to A_1, A_2, A_3, and A_4.



(b) **No**. A complete graph is one where every pair of vertices is connected by an edge. Clearly A_1 and A_3 are not connected. Hence, in this case the graph is not a complete graph.
(c) The adjacency matrix M is as follows,

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

4. A sum of INR 12,000 is invested in two parts: one part earns **simple interest** at 8% per annum, and the other earns **compound interest** at 6% per annum, compounded yearly. After 2 years, the total interest earned from both parts is INR 1,847.20 . How much was invested in each part?

(12)

SOLUTION P_1 = INR 10,000 and **P_2 = INR 2,000.**

We need to split INR 12,000 into two parts, **P_1** (simple interest) and **P_2** (compound interest), such that **P_1 + P_2 = 12000**. Total interest from both parts is INR 1,847.20 we have **$0.16P_1 + 0.1236P_2 = 1847.2$**. Note that, the steps involving appropriate formulas for simple interest and compound interest is needed. Hence, we have two equations in two variables, solving which shall fetch us **P_1 = 10000** and **P_2 = 2000.**

End of Quiz. Good Luck!

Ashoka University

Monsoon 2024

FC-0306-4

Quantitative Reasoning and Mathematical Thinking

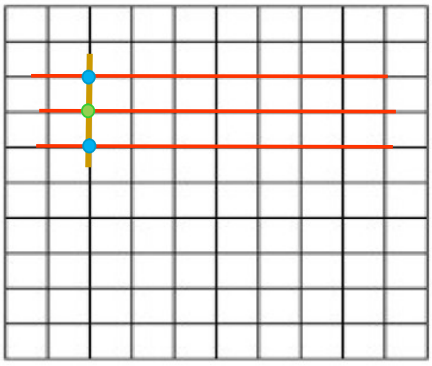
Date: 30 September 2024 **Total points: 100** (= 80 + 20 (Bonus)); Credit: 25%

Mid-Term Test **Time:** 4:50 PM - 6:10 PM

Name: ..... **Ashoka ID #** .....

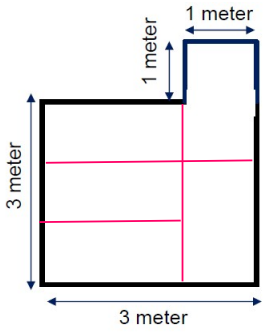
Instructions: This is an MCQ or short-answer type, Restricted OPEN-NOTES, quiz. For each **MCQ**, please choose one answer from the given choices. Each *correct answer* to an MCQ will fetch 4.0 points, *incorrect answer* will contribute 0 point, and *no answer* (omission) leads to 1.0 point. For a short-answer type question, its components, if any, will be graded proportionately. Unless otherwise mentioned, all questions carry **4 points**, each. This quiz has **seventeen** questions and **three** pages.

Submission of answers: Please write **your name, ID**, and your **answers** on this Test Paper itself.



1. Consider an (11 × 11) rectangular grid as shown in the figure on the left. Each grid point (intersection of horizontal and vertical lines) is colored randomly with either of two colors B (blue) or G (green). Prove that there always exists a rectangle R with its four corners lying on grid points all of which are colored with the same color. **(8 points)**

Solution: Consider any three horizontal grid-lines A, B, C and a vertical line L that intersects them. Each of the three intersection points on L is colored with blue or green such as BBB, BBG, BGB, …, BGG, GGG. So, there are 2^3 = 8 possible color assignments (bands). If we consider nine such vertical lines, then by pigeon-hole principle, there exist at least two vertical lines intersecting with A, B, C with the same color band. Also, since there are two colors for coloring three intersection points on a vertical line, by pigeon-hole principle again, at least two intersection points must have the same color. Hence, there will be at least two vertical lines whose all four intersection points with at least two horizontal lines, must be of the same color, proving the claim.



2. Six small insects are located randomly in an L-shaped room as shown in the figure on the left. Show that there always exist two insects, the separation of which is at most √5 meter. **(8 points)**

Solution: The room can be partitioned into five rectangles, each with dimension (2 meter × 1 meter) as shown. By PHP, there must exist at least one rectangle with two insects. As the diagonal in such a rectangle is of length √5 meter, the claim follows.

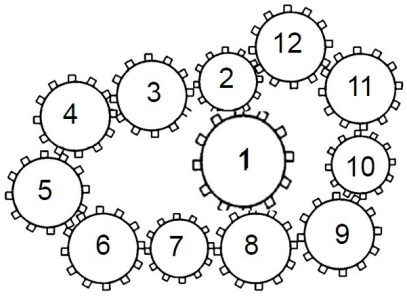
3. From the alphabet $\{A, B, C, \ldots, Y, Z\}$, we want to construct three-letter strings such that letters are alphabetically ordered from the left with repetition allowed, and the last letter is always Z. For

example, *BBZ* and *CMZ* are valid strings, whereas *MCZ* or *CMY* or *CCY* are not. The total number of such valid strings is (*choose one*):

A. 326; B. 650; C. 300; D. 325; E. None of these.
(Solution: 351)

4. 101 students have a total of Rs. 5000/- (Rs. Five thousand). Mark among the following statements, the correct one:

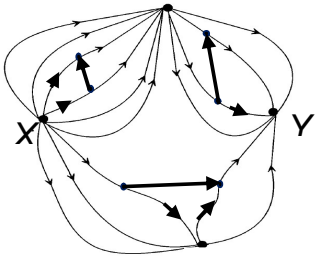
A. All of them may have different amount of money;
B. At least two of them must have the same amount of money;
C. At least three of them must have the same amount of money;
D. All of them, except one, must have the same amount of money;
E. The distribution of money could be random and nothing can be predicted.



5. Consider the gear assembly as shown in the figure on the left. Gear 1 is the driving gear and it is rotating in anti-clockwise direction. Mark among the following statements, the correct one:

A. Both Gear 4 and Gear 11 will rotate clockwise;
B. Gear 4 and Gear 7 will rotate in the same direction;
C. Gear 6 and Gear 9 will rotate in the same direction;
D. Gear 4 and Gear 9 will rotate in the same direction;
E. None of the above.

6. The road network from City *X* to City *Y* is as in the figure on the right. Traffic is allowed only along the directions shown. The number of possible routes for traveling from *X* to *Y* is (choose one):



A. 32 B. 31 C. 41 D. 42
E. None of these

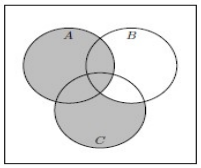
7. A ceramic coffee cup is given to you. What are the reasons behind its existence as per Aristotle's enquiry on the Laws of Causality?

1. *Causa materialis*: the clay mixture; 2. *Causa formulis:* the cup design; 3. *Causa efficiens*: the potter with the knowledge of pottery; 4. *Causa finalis*: for the purpose of drinking.

8. For three sets *A*, *B*, and *C*, if $A \subseteq B$ and $B \subseteq C^C$, then prove using set algebra that $A \cap C = \varphi$.

Solution: Let $x \in A$; then $x \in B$ and also $x \in C^C$, i.e., $x \notin C$. This will be true for any $x \in A$; Hence, $A \cap C = \varphi$.

9. Write down the set expression for the *shaded area* in terms of *A*, *B*, *C*:



Solution: $A \cup [B^C \cap C]$; there are many other ways of writing it.

10. Let *X* be the sequence: (1, 2, 3, …., 99, 100). A subsequence of *X* is a selection of elements from *X* preserving the increasing order. For example, (2, 5, 27, 99) is a valid subsequence but (47, 10, 51, 93, 99) is not. The number of distinct non-empty subsequences is (fill-up the gap): $2^{100} - 1$.

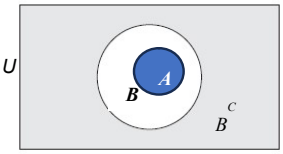
11. Consider the set X of all 3-digit palindromic numbers constructed over the digits: (1, 2, 3, 4, 5, 6, 7, 8, 9). The cardinality of the power set $P(X)$ of X is (fill-up the gap): 2^{81}.

12. Let X be the set: {1, 2, 3, …., 99, 100}. A valid 2-subset is a set $\{a, b\}$ where $a, b \in X$ and the difference between a and b is greater than or equal to 2. For example, {10, 14} is a valid 2-subset but {5, 6} is not. The number of valid 2-subsets of X is (choose one):

A. 4950; B. 2475 ; C. 5050; D. 4851; E. None of these.

13. Assume A and B are two sets. Prove using Venn diagram that $A \subseteq B$ *if and only if* $A \cap B^C = \emptyset$.

Solution: *If part:* Let $A \cap B^C = \emptyset$. As seen from the Venn diagram, No part of A can intersect with B^C. Hence, $A \subseteq B$.
Only if part: L et $A \subseteq B$ as in the Venn diagram. Clearly, $A \cap B^C = \emptyset$.

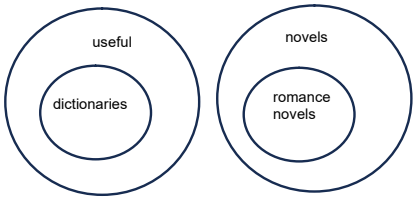


14. Let X be the set of all positive integers that are multiples of 5, i.e., $X = \{5, 10, 15, 20, 25, \ldots\}$. Prove that X is countably infinite. **(8 points)**

Solution: Denote by N, the set of natural numbers. Let f: $N \to X$ such that $f(x) = 5x$. Thus, there is a bijection between N and X. Hence, X is countably infinite.

15. Consider the following statements:
$S1$: All dictionaries are useful; $S2$: Pallavi owns romance novels; $S3$: No novel is useful.
Use a Venn diagram to determine whether or not the following conclusion C is correct:
C: Pallavi may own a dictionary as well.

Solution: The Venn diagram is shown here. Since, Pallavi may also own books other than romance novel, she may own a dictionary as well. Hence C is correct.



16. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ be the universal set. Construct three sets A, B, C to prove that $A \cup (C \setminus B) \neq (A \cup C) \setminus B$.

Solution: Let $A = \{1, 4, 5, 7\}$; $B = \{2, 4, 6, 7\}$; $C = \{3, 5, 6, 7\}$;
So, $C \setminus B = \{3, 5\}$; $A \cup (C \setminus B) = \{1, 3, 4, 5, 7\}$;
$(A \cup C) = \{1, 3, 4, 5, 6, 7\}$; $(A \cup C) \setminus B = \{1, 3, 5\}$; Hence, $A \cup (C \setminus B) \neq (A \cup C) \setminus B$
There are many other solutions as well.

17. In a batch of 200 students, 50 students took the course on History, 90 students took the course on Philosophy, and 80 students took the English course. 20 students took both History and English; 35 students took both History and Philosophy; 40 students took both Philosophy and English. 15 students took all three courses. The number of students who took only History is (*choose one*):

A. 10; B. 15 ; C. 20; D. 35; E. None of these.

End of Test. Good Luck!

Ashoka University

Monsoon 2024

FC-0306-4

Quantitative Reasoning and Mathematical Thinking

Date: 09 December 2024 **Total points: 100 (**= 80 + 20 (Bonus)); Credit: 25%

End-Sem Test – Model Solution **Time:** 9:00 AM - 10:30 AM

Name: ..... **Ashoka ID #** .....

Instructions: This is an MCQ or short-answer type, OPEN BOOK, OPEN-NOTES, quiz. For each MCQ, please choose one answer from the given choices. Each *correct answer* to an MCQ will fetch 4 marks, *incorrect answer* will contribute 0 mark, and *no answer* (omission) leads to 1 mark. For a short-answer type question, its components, if any, will be graded proportionately. This quiz has **fourteen** questions and **three** pages.

Submission of answers: Please write **your name, ID**, and your **answers** on this Test Paper itself.

	A	B	C	D	E	F	G	H
A	0	1	0	0	0	0	0	0
B	1	0	1	0	1	0	0	0
C	0	1	0	0	0	0	0	0
D	0	0	0	1	1	0	0	0
E	0	1	0	1	0	0	0	0
F	0	0	0	0	0	0	1	1
G	0	0	0	0	0	1	1	0
H	0	0	0	0	0	1	0	0

1. **(8 marks)** Consider the following adjacency matrix for an undirected graph *G* and answer the following questions:

(a) Which of the following is the most appropriate description of the category of graphs to which *G* belongs (please put tick mark)?

A. Simple graph
B. Graph with parallel edges
C. Graph with self-loop
D. Both B and C
E. None of these;

(b) Does there exist a path from vertex *B* to vertex *D*? Yes/No

(c) Is *G* disconnected? Yes/No (4 + 2 + 2)

2. **(4 marks)** Consider the permutations P1: (1 2 3 4 5 6) → (4 1 6 5 2 3) and P2: (1 2 3 4 5 6) → (2 3 4 1 6 5). Your friend Pallavi, who is an expert in graph theory, tells you that P1 and P2 are very similar. Explain just in *one sentence* the basis for her comment.

Ans: Consider the directed graph representation of permutations; P1: 1→ 4 →5 → 2 →1, and 3→6→3; P2; 1→ 2 →3 → 4 →1, and 5→6→5; Thus, both P1 and P2 comprise a 4-cycle and a 2-cycle.

3. **(4 marks)** Write in symbolic logic the following statement: "Every student in the QRMT class has a cell phone."

Ans: Let let X denote the set of all QRMT students; $P(x)$ denote "x is a student in QRMT", and $Q(x)$: "x has a cell phone". Then the given statement can be symbolically written as: $\forall x \in X,\ P(x) \Rightarrow Q(x)$; Alternatively, the given statement can be written as: $\forall x \in X,\ Q(x)$.

4. **(4 marks)** Write the inverse of the following statement: "If it rains today, I will take my umbrella."

Ans. Inverse: If it is does not rain today, I will not take my umbrella (inverse of implication $P \Rightarrow Q$ is $\neg P \Rightarrow \neg Q$).

5. **(4 marks)** Let P, Q, R, S be four statements such that $P \Rightarrow Q \Rightarrow R \Rightarrow S \Rightarrow P$. Then, which of the following statements are correct?

A. $P \Rightarrow S$ B. $R \Rightarrow Q$ C. $P \Leftrightarrow S$ D. $\sim P \Rightarrow \sim S$ E. None of A, B, C, D is TRUE.
Answer: A, B, C, D are correct.

6. **(8 marks)** Label each set below as empty or non-empty:
A. $\{x : x$ is an even number and $(x + x^2 + 1)$ is even$\}$; Empty
B. $\{x : x$ and $x + 1$ are relatively prime, x is a positive integer $> 1\}$. Non-Empty (4 + 4)

7. **(8 marks)** Let P and Q denote the statements: 'The election is decided' and 'The votes have been counted', respectively. Express each of the following English sentences with symbolic logic:
(a) If the votes have been counted then the election is decided, and conversely; $P \Leftrightarrow Q$
(b) The votes have been counted unless the election is not decided; $P \Rightarrow Q$
(c) The votes have been counted whenever the election is decided; $P \Rightarrow Q$
(d) The election is decided when the votes have been counted. $Q \Rightarrow P$ (2 + 2 + 2 + 2)

8. **(4 marks)** Consider the following algorithm in pseudocode:

Procedure OP(a, b)
integer: a, b
while $a \neq b$
if $a > b$
$a := a - b$
else $b := b - a$
return OP(a, b) = a

The value returned by OP(12, 143) is (choose one):

A. 1 ; B. 4 ; C. 6; D. 12; E. None of these.

9. **(8 marks)** Let A and B denote two positive integers, where
$A = 2^{\alpha} \times 3^{\beta} \times 11^{\gamma} \times 17^{\beta}$; $B = 2^{\beta} \times 3^{\alpha} \times 5^{\gamma} \times 17^{\delta}$; $\alpha, \beta, \gamma, \delta$ are positive integers, $\alpha < \beta < \gamma < \delta$.
Then (fill-up-the-blanks):

(a) The GCD(A, B) = $2^{\alpha} \times 3^{\alpha} \times 17^{\beta}$;

(b) The number of distinct factors in $(A \times B)$ is: $(\alpha + \beta + 1)^2 \times (\gamma + 1)^2 \times (\beta + \delta + 1)$; (4 + 4)

10. **(4 marks)** A computer can only multiply two numbers at a time. The minimum number of multiplication operations a computer would take to compute z^{23} using the binary-based Exponential Algorithm, where z is an integer, is (choose one):

A. 3 ; B. 4 ; C. 5; D. 6; E. None of these.

11. **(8 marks)** 100 distinct points are randomly scattered on the 2D-plane such that the distance of each of them from the origin is distinct. We want to draw a circle that passes through one of these points and encloses exactly 1/3 of the remaining points in its interior. Your friend Pallavi, who is also an expert in algorithms, tells you that just after the k-th iteration of Bubble Sorting, a solution to this problem can be obtained.

(a) The minimum value of k as told by Pallavi, is possibly (choose one):

A. 3; B. 30; C. 67; D. 34; E. None of these.

(c) Justify your choice above using a single sentence.

Ans. Bubble-sort can be used to sort the radial distances in the *ascending order*, and in the 34^{th} iteration, it will find the distance (d) of a point, which is greater than those of 33 points; hence a circle centered at the origin and with radius d, will enclose 33 points, i.e., 1/3 of the remaining points. (4 + 4)

12. (**8 marks**) You are given a ruler (unmarked), a collapsible compass, and a straight-line segment of unit length. Now, among the following statements, *mark those* which are **correct**:

A: Given two line segments P and Q intersecting at an angle α, it is possible to draw two other line segments R and S intersecting at an angle β such that $\alpha = \beta$;

B: It is possible to construct a line segment of length $\sqrt{2}/\sqrt{5}$;

C: A triangle with a side length = 2, and with two angles 30°, 10° incident on its end-points, is constructible;

D: A regular polygon with seven sides, each with unit length, is constructible;

E: A ruler plus two non-collapsible compasses is more powerful than a ruler plus one collapsible compass, as far as the constructability is concerned.

Answer: A and B are correct.

13. **(4 marks)** Today's earthquake in Surat is 8 whereas that in Kathmandu is 5, both measured in the Richter scale. Then the devastation in Surat is likely to be …1000…….. times* more powerful than that in Kathmandu (**Fill-up the blank**).

* in terms of magnitude (Richter scale usually refers to the measure of magnitude); however, in terms of total energy, one may say that it is 32^3.

14. **(4 marks)** Between two cities P and Q, there are two *disjoint* roads. There are two bridges A, B on one road, and two bridges C and D on the other. A motorist takes around 1 hour to travel from P to Q via either of the roads. The probability of each bridge being shut for repair today is ½. Then the probability of a motorist to reach Q starting from P today, without stopping at any bridge, is (choose one):

A. ½ ; B. 1/16 ; C. 7/16; D. 1/4; E. Incomplete data.

End of Test. Good Luck!